Metaphor in Mathematics & Sound

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Invited talk at Matematica e Cultura 2006

Universitá di Ca' Foscari

24-26 March 2006

Applications of mathematics in sound

- Composers have always drawn inspiration from mathematical ideas
- 21st century composers make extensive use of mathematical tools in their work.

Parallels between mathematical & musical *thinking*

- ✓ Mathematicians and composers make use of many of the same metaphors as a tools for nonverbal communication, understanding, reasoning, discovery, and creation

How is it possible to convey meaning without words?

- CS Peirce Semiotic: 3 kinds of “signs”:
  - Icon — imitative
  - Symbol — icon that requires a common culture
  - Index — cross-domain mapping

A sad story

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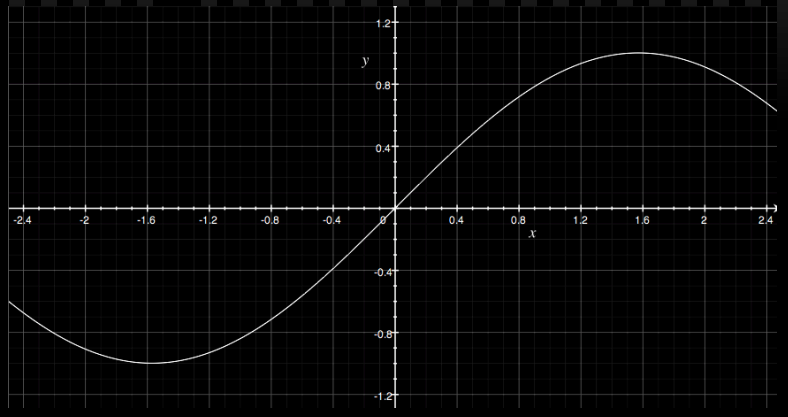


Data Sonification is a sound index

- Mystery Sound 
  - Cross domain inference-preserving mapping
    - 0th order mapping
      - Data as audio signal
    - 1st order
      - Audio signal is the carrier
      - Data is the modulator

Index & Conceptual Metaphor

- Lakoff & Johnson's *Conceptual Metaphor*
  - Cross domain, inference preserving mapping
- Theory of embodied mind
  - Some concepts are innate
  - Some concepts we learn through the body's recurring patterns of dynamic interaction with the physical world
    - Generalized as Schemata
  - The rest we understand and communicate by means of comparisons to those basic recurring patterns
    - Conceptual Metaphors

Metaphor in Sound & Mathematics

- Lakoff, Núñez, Johnson
  - Linguistics, mathematics, visual arts
- Extend the idea to sound
- Show:
  - Mathematicians & composers use many of the same metaphors when reasoning about and communicating abstract concepts

Three (of the many) Patterns

- ✓ Container
- ✓ Force
- ✓ Path

Container

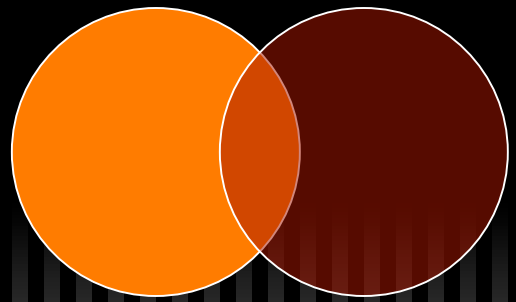
- Containers are ubiquitous
  - We are in a room, the room is in a building
  - Inside the room are pens containing ink or glasses containing water
  - We ourselves are containers
- Space or Object (depending on your POV)
  - Inside: a Space
  - Outside: an object

Linguistic Inside/Outside Metaphors

- Restriction
  - He is an *out*law
  - operando *al di fuori* delle regole
  - *en dehors* de la loi
- Category or a State
  - Lui è un po' *di fuori*.
  - You must be *out* of your mind.
  - He was deep *in* thought

Mathematics Container

- A Set as a Container
  - Reasoning with Venn diagrams
- Closure
  - An operation on elements in a set results in elements that are still *inside* that set



Mathematics Inside the Container

- Space
  - Vector space, State space, Affine space
  - Distance metrics (closeness of two elements)
  - Intuitive vs mathematical space
    - Intuitive: elements are *in* the space
    - Mathematical: Space *is* a Set of Elements
      - Properties of the Space: relations between members of the set

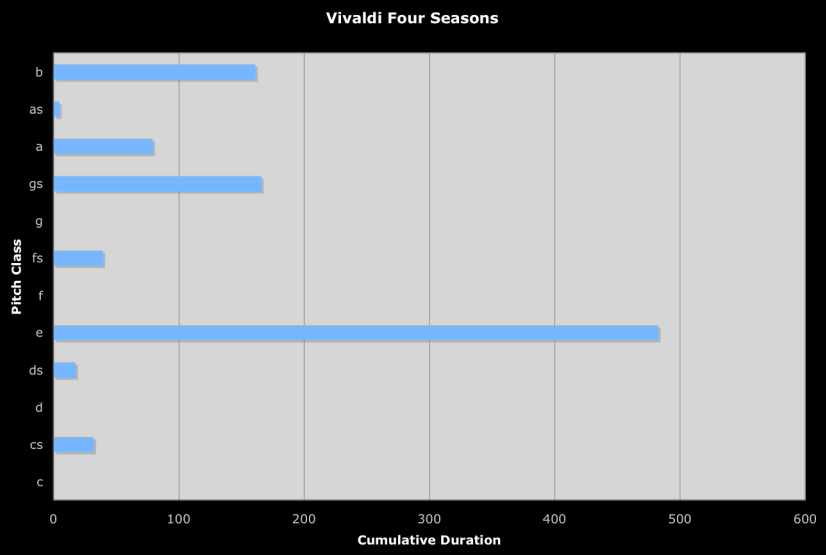
Sound Container (literal)

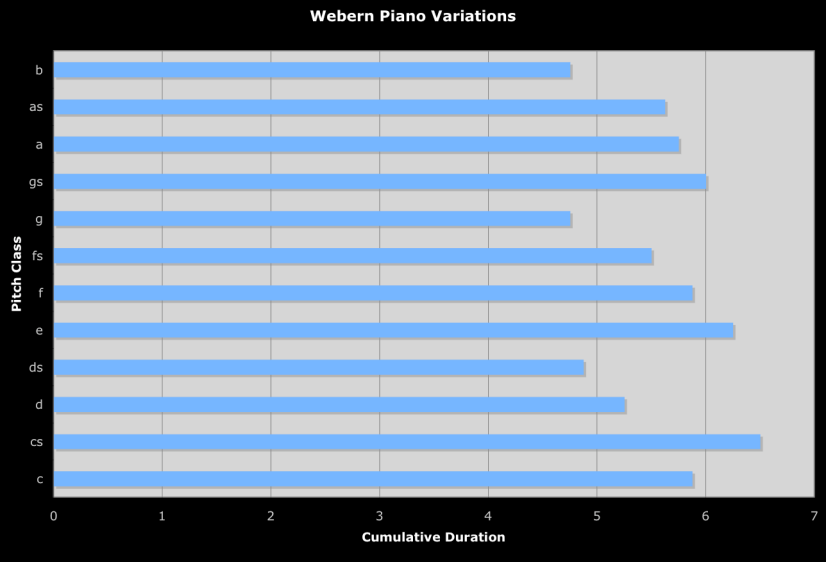
- Inside: Reverberation
  - Tuva singer in the Earth 
- Outside: formants or modes
  - Size implied by frequency of formants or modes
    - Large, dangerous 
    - Harmless, cute 

Sound Container (abstract)

- A key or a raga feels like a space
  - Points (note events)
  - Structure (relations between notes)
    - Characteristic patterns
    - Emphasis by duration, accent

A space defined by cumulative duration of pitch class





Force (gravity)

- Down requires less energy than up
  - Wake *up*. *Fall* asleep
  - My computer is *down*. It *crashed*.
  - Sono *giù*
  - *Debout* la République
  - We felt *weighed down* by the *gravity* of the situation
- We associate MORE with UP
  - Containers under gravity
    - As you add things to a pile its top gets higher
    - As you fill a glass, the level of liquid in it gets higher
  - Linguistic: More is up
    - a *basso* prezzo
    - Turn *up* the volume

Mathematics: More is up

- Graphing functions
  - Larger values towards the "top" of the screen
  - Or “top” of a sheet of paper

Sound: More is up

- *Higher* pitches (*more* cycles per second)
  - Internalized muscle tension
    - Requires more energy to produce with the voice
- Higher frequency, higher amplitude value, higher rates of events, higher density of events
  - Metaphor for *More* tension, energy
  - And the converse

Linguistic Force variants

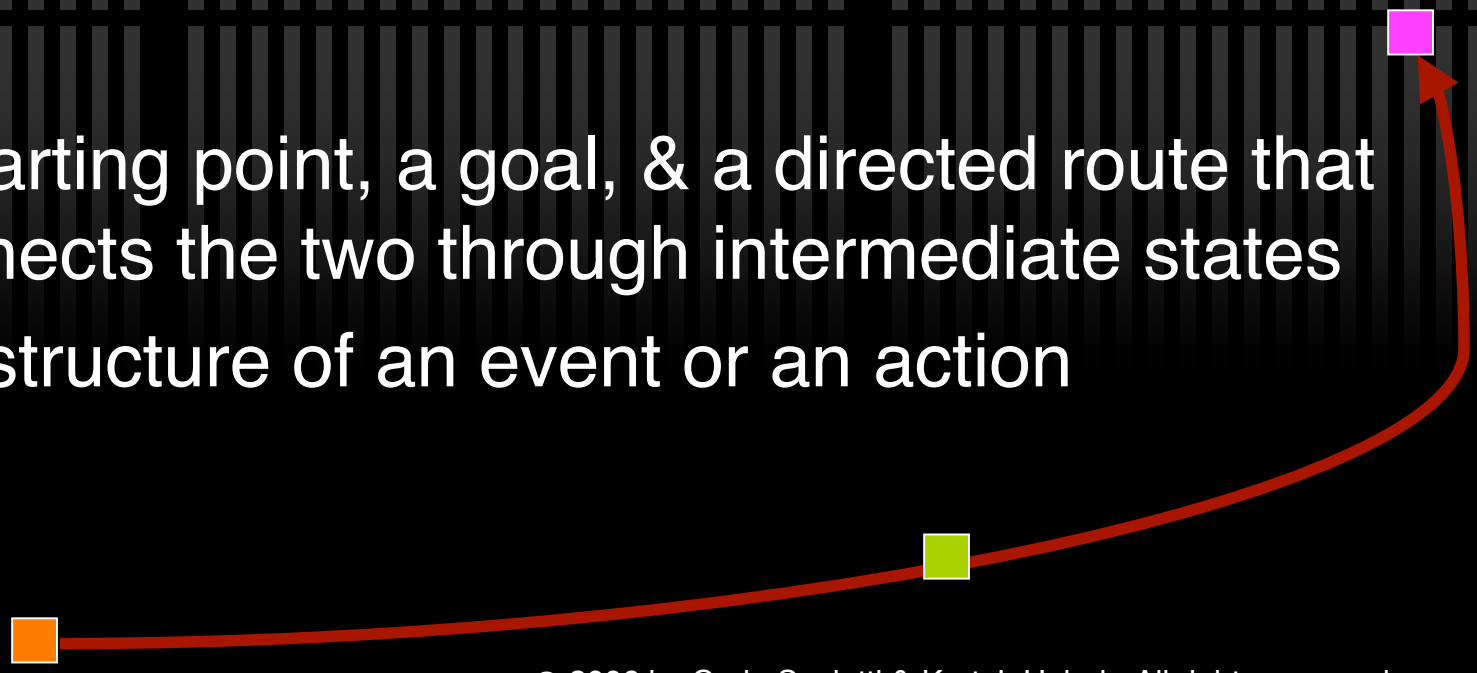
- *Impact* of an idea or realization
  - un *coup* de tête
  - I was gob*smacked*
  - J'ai pris une *claque*!
  - And then it *hit* me (sudden understanding)
- Power of sexual attraction
  - C'est une vraie *bombe*!
  - He has a *magnetic* personality
  - il *colpo di fulmine*

Impact & Attraction

- Mathematics
  - Dynamical systems have
    - Attractors
    - Domains or basins of attraction
- Sound
  - Attraction
    - Sense of getting into the groove (a gravity well)
  - Striking or displacing an object
  - More force == louder (also more harmonics)

Path

- Force causes movement of an object with mass through space in a direction
  - Initiates a causal sequence (which leads us to Path)
- Path
  - A starting point, a goal, & a directed route that connects the two through intermediate states
  - the structure of an event or an action



Linguistic path

- *Follow* the *steps* of a logical argument
  - Or *Take a leap* of faith
- ça *va*? come *va*? how's it *going*?
- The story *meanders* for a while without ever getting *anywhere*
- The *road* climbs through a series of switchbacks until *it* finally reaches the summit

Mathematical Proof as a Path

- A direct proof
  - A path or sequence of claims from axiom through intermediate true statements to arrive at a final result
  - Even the symbolic notation implies a path
    - P —> Q

Other Mathematical Paths

- Dynamical System
  - A means of describing how one state develops into another state over the course of time
    - Eric W. Weisstein. "Dynamical System." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/DynamicalSystem.html
  - *Trajectory*
    - depends on Initial conditions
    - *passes through regions* of state *space*
- Functions as paths
  - The function *starts* at -1 and *ends* up at 3 so it must have *passed through* zero

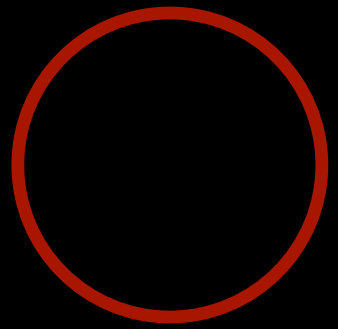
Sound metaphors for path

- Literal movement of sound sources
- Movement through abstract spaces
  - Modulation
    - from one key (state) to another
    - from one meter to another
  - Path through a parameter space
    - Accelerando, crescendo, glissando

Beethoven Op 127 Finale

Orange Answerfax

Cycle



- A repeating action or event in which
  - the end of one path is the start of the next
  - start and end states are in some way equivalent
- ✓ A circle, a sinusoid or spiral
  - ✓ Heartbeat, breathing
  - ✓ Sleeping/waking
  - ✓ Mental obsession

Infinity

- *Potential* infinity
  - Ongoing process that will continue indefinitely
    - Iterative or continuitive process with an intermediate result
      - Continuitive (flying)
      - Iterative (jumping)
- *Actual* Infinity
  - Potential infinity PLUS a (metaphorical) terminal state
    - Initial state
    - Iterative process with indefinite number of iterations
    - A resultant state after each iteration
    - PLUS a final state

In language

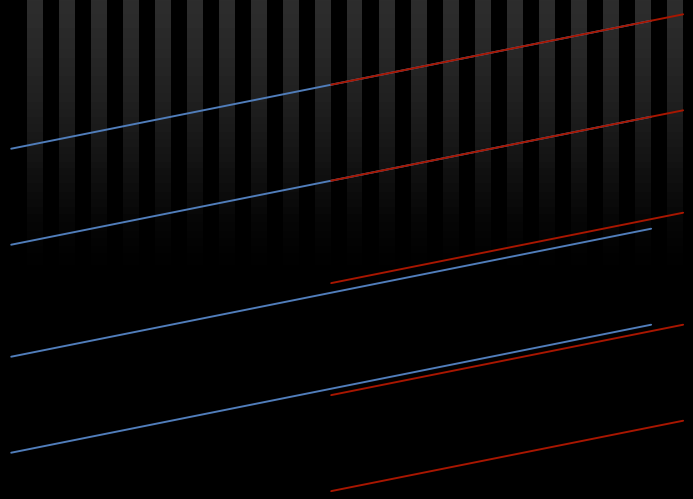
- Potential infinity
  - <verb> and <verb> and <verb>
  - The boy jumped and jumped and jumped again
  - Le chat court et court et court
- Actual infinity
  - The sun burned and burned and burned *forever*.
  - And they lived happily *ever* after

Mathematics

- Limits
  - e.g. a bouncing ball loses half its height on each bounce
    - Start: 1
    - Process: Divide in half
    - Intermediate result: $x_n = 0.5 * x_{n-1}$
    - In the limit (after infinite time): 0
- Proof by induction
    - Start (Basis): Show that it is true for 1
    - Process: (Inductive step): If it is true for n=m then it is true for n = m+1
    - Conclusion: If it is true for a *starting* value and the *process* is valid, then it is true for *all* natural numbers

Sound of potential infinity

- Shepard tones & rhythms
  - Start: at a frequency: f
  - Process: Continuous (or discrete): $f_n = f_{n-1} * 2^{i/12}$
  - Start new one when 1st reaches equivalent state



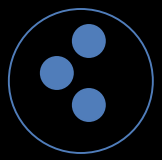
Perceptual limits

- Perceptual limit as metaphor for final state
  - The infamous “Fade out” to end a repetitive song
    - Amplitude drops below threshold of perception…
  - Frequency can get higher and higher
    - And disappears out of range of perception
  - Click repeats faster and faster
    - And becomes a pitch in the limit
    - Exponential accelerando
      - Metaphor for theoretical limit on bounce

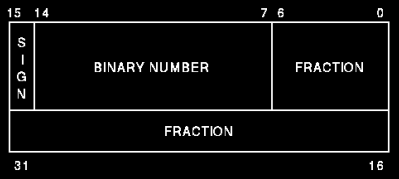
Metaphor can illuminate (or obscure)

- Metaphors within mathematics
  - Mapping a problem in one domain into an equivalent problem in another domain
    - Algebra -> topology
    - Equivalence class of NP complete problems
    - Mapping an unsolved problem into a domain where you already know the solution

New metaphor reveals new numbers

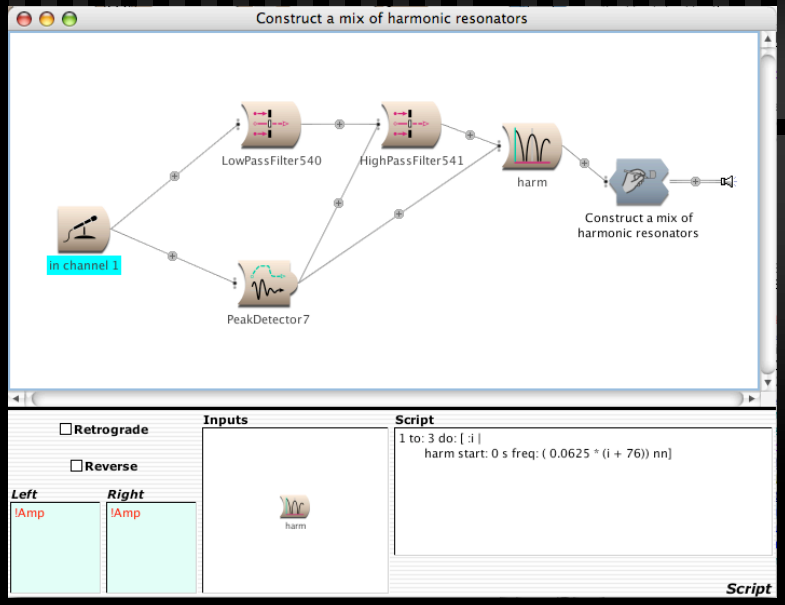


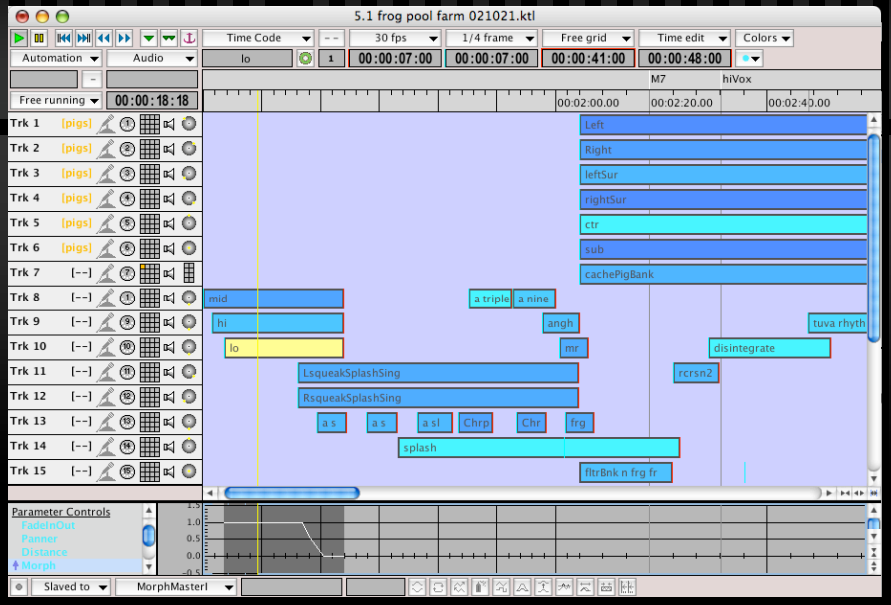
- Number as a collection of objects in a container
  - Add/Subtract by putting in or removing objects
  - Natural numbers (but not fractions)
- Number as a length of a string
  - Add/Subtract by laying strings end-to-end, folding, cutting
  - Suggests irrational numbers (but not 0 or negatives)
    - arrange strings as right triangle or circle
- Floating point metaphor
  - Influential over mathematics, science, culture due to pervasiveness of computers
    - Two representations of 0
    - Cannot represent irrational numbers like pi
    - Associative property is not necessarily true
      - Limited precision
      - $(\delta + \varepsilon) + H \neq \delta + (\varepsilon + H)$



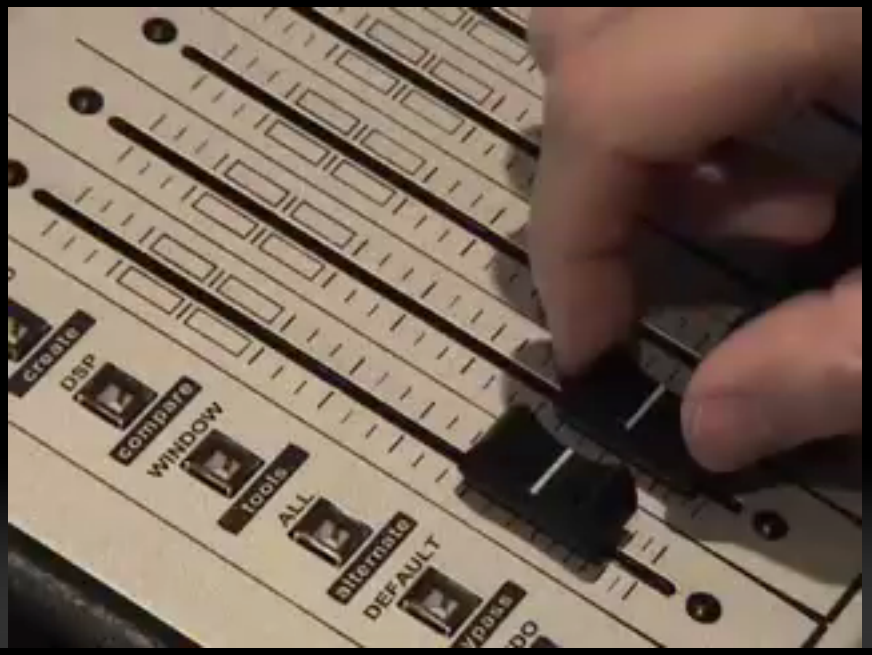
Computer-Human Interface metaphor

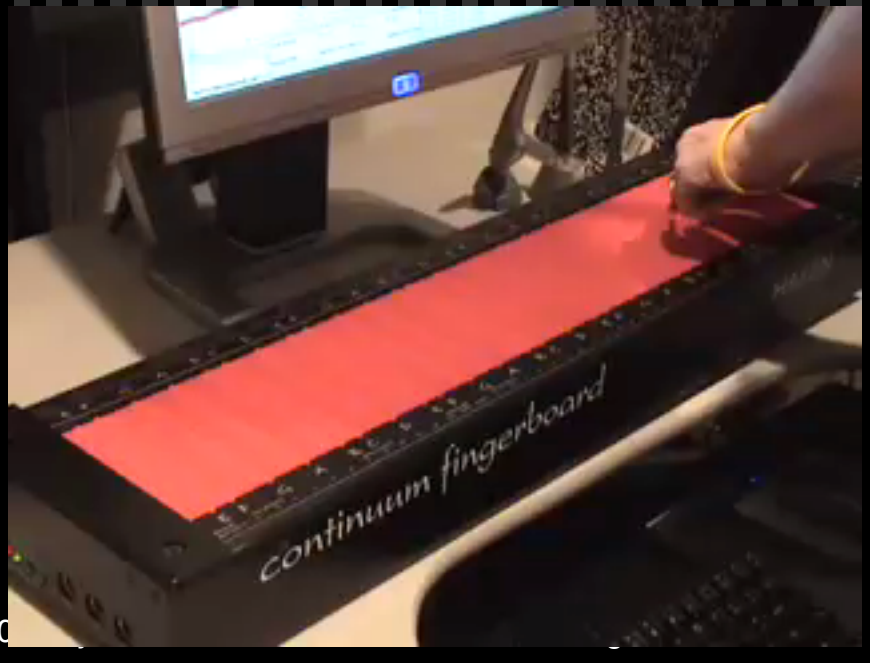
- ✓ Kyma Signal flow & Timeline





Controller as Metaphor for sound parameter space





Metaphor implies set of constraints

- Playing within a set of rules
  - Solving a puzzle
- Systematically breaking a rule
  - A What-If Scenario
    - A new kind of geometry?
    - A new kind of music?

Conclusion

- In Mathematics and Sound we use metaphors to reason with (manipulate) abstract ideas in nonverbal ways & to communicate those concepts to others
  - Guide their respective audiences along a path through abstract spaces experienced via analogy to physical space and orientation
- Choice of metaphor
  - can illuminate or obscure the abstract concept
  - Imply what kind of mathematics or music will follow

The End